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$$\text{R} = h^2(1+u^2)\left(u+\frac{d^2u}{d\omega^2}\right),$$

in which u is the cotangent of ρ.

The analogy between the formulæ given in this paper and those usually employed in discussing the motion of a point on a plane is very striking. The former too become identical with the latter when the portion of the sphere on which the trajectory is described becomes infinitely small in comparison with the radius.

The Rev. H. Lloyd V. P. read the following paper " on a New Magnetical Instrument, for the Measurement of the Inclination, and its Changes."

In order to know all that relates to the earth's magnetic force, at a given place, observation must furnish the values of three elements. Those which naturally present themselves for immediate determination are, the *intensity* of the force itself, and the two angles (the *declination* and *inclination*) which determine its direction. We may substitute for these, however, any other system of elements which are connected with them by known relations. Thus, we have hitherto preferred to observe the *declination*, and the *two components* (horizontal and vertical) *of the intensity;* and, in general, the main considerations which should guide us in our choice are, the exactness of the observed results, and the facility of their determination.

In this point of view, the *declination* and the *horizontal component of the intensity* leave us nothing to desire, their determination being now reduced to a degree of precision, hardly (if at all) inferior to that of astronomical measurements. The same thing, however, cannot be said respecting the third element, as hitherto observed. In the Dublin Magnetical Observatory, and in the Observatories since established by order of the Government and of the East India Company upon the same plan, the third element chosen for observation has been the *vertical component of the intensity*, the instrument for the measurement of which has been already submitted to the notice of the Academy. The principle of this instrument, it will be remembered, is to balance the vertical component of the magnetic force by a fixed weight, and to observe the changes of the position of equilibrium, under the action of the changing force. Unexceptionable as this principle is in theory, the accuracy of the results has not been commensurate with that of the other two instruments. This inferiority is to be traced to the large influence which the unavoidable errors of workmanship must necessarily have on the position of equilibrium of a magnet supported on a fixed axle. It has been shown that the effect of magnetizing a bar, under the most advantageous circumstances of form, and at the part of the globe where the vertical component of the magnetic force is greatest, is the same (as to its position of equilibrium) as if its centre of gravity had been transferred about the $\frac{1}{40}$th of an inch towards the north end; so that the moment of the force, exerted by the vertical component of the earth's magnetism, can never exceed this small quantity multiplied by the weight of the bar. Now, in order to render the results of this instrument comparable to those of the horizontal-force magnetometer, it should enable us to measure changes of the vertical force, amounting to the $\frac{4}{100,000}$dth part of the whole; i. e. we have to measure effects, such as would be produced by shifting the centre of gravity through the *one-millionth of an inch*. It will be easily understood, from this statement, how great must be the effect of a minute disturbance of the relative parts of the instrument, or of inequalities in the bearing points of the axle; and experience has accordingly shown that it is altogether unavailable for the determination of changes of long period.

The same difficulties, and from the same source, have been found to attach to the usual method of observing the *magnetic inclination*, and its changes, however refined the construction of the instrument. The sources of error seem, in fact, to be inherent in every direct process of determining the third element; and it is only by an *indirect* method that we can hope to evade them.* Of this character is the method now proposed.

If a soft iron bar, perfectly devoid of magnetic polarity, be held in a vertical position, it immediately becomes a temporary magnet under the inducing action of the earth's magnetic force, the lower extremity becoming a north pole, and the upper a south pole. Accordingly, if a freely-suspended horizontal magnet, whose dimensions are small in comparison with those of the bar, be situated near, in a plane passing through one of these poles, it will be deflected from the magnetic meridian. The deflecting force is the induced force of the bar, which may be regarded as proportional to the energy of the inducing cause, i. e. to the *vertical component* of the earth's force; while the counteracting force is the *horizontal component* of the same force, acting directly on the magnet itself, to bring it back to the magnetic meridian. Thus the magnet will take up a position of equilibrium, under the action of these opposing forces; and this position will serve to determine the ratio which subsists between them. When the right line connecting the centre of the horizontal magnet, and the acting pole of the bar, is perpendicular to the magnetic meridian, the tangent of the angle of deflection will measure the ratio of the two forces, and will therefore be proportional to the *tangent of the magnetic inclination*. Accordingly, by observing the changes of position of the horizontal magnet, so circumstanced, we can infer those of the inclination itself.

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* Two such indirect methods of determining the inclination have been proposed in Germany, one by Professors Gauss and Weber, the other by Dr. Sartorius von Walterhausen. That now suggested bears a close analogy, in principle, to the former of these: it differs from it, however, not only in the means employed, but also in the end in view,—the main object of the present method being the determination of the inclination-changes.

But the iron bar may have (and generally will have) a certain portion of *permanent* magnetism, which will concur with the induced magnetism in producing the deflection; and it becomes necessary to institute the observations in such a manner, as to be able to eliminate the effects of this extraneous cause. For this purpose we have only to invert the bar, so that the acting pole, which was uppermost in one part of the observation, shall be lowermost in the other. The induced polarity will, under these circumstances, be opposite in the two cases; and the acting force will in one case be the *sum* of the *induced* and *permanent* forces, and in the other their *difference*.

Let X and Y denote the horizontal and vertical components of the earth's magnetic force, M the intensity of the permanent magnetism in the acting pole, and m the magnetic moment of the suspended magnet. The intensity of the induced magnetism is, by hypothesis, equal to

$$k\mathrm{Y},$$

k being an unknown constant; and when this is of the same name as the permanent magnetism, the intensity of the acting force, at the unit of distance, is

$$k\mathrm{Y} + \mathrm{M}.$$

Accordingly, the moment of this force to turn the suspended magnet is $(k\mathrm{Y} + \mathrm{M})\,mr\cos u$, u being the angle of deflection, and r a constant depending on the distance; or, making, for abridgment, $kr = p$, $\mathrm{M}r = q$,

$$(p\mathrm{Y} + q)\,m\cos u.$$

But this deflecting force is resisted by the earth's horizontal force, the moment of which to turn the magnet is

$$\mathrm{X}m\sin u\,;$$

and the magnet will rest when these moments are equal. Hence the equation of equilibrium is

$$p\text{Y} + q = \text{X} \tan u. \qquad (1)$$

By the same reasoning it will appear, that when the induced and permanent magnetisms are of contrary names, there is

$$p\text{Y} - q = \text{X} \tan u'; \qquad (2)$$

in which u' is the new angle of deflection when the bar is inverted. And adding these equations together, and observing that $\text{Y} = \text{X} \tan \theta$, θ being the inclination, we have

$$2p \tan \theta = \tan u + \tan u'. \qquad (3)$$

This equation would furnish at once the inclination sought, provided we knew the value of the constant k. In order to determine it, we have only to place the iron bar *horizontally* in the magnetic meridian, its acting pole remaining in the same place as before, but pointing alternately to the north and south. The inducing force is, in this case, the *horizontal component* of the earth's magnetic force; and it will be readily seen that the equations of equilibrium are similar to (1) and (2), substituting X for Y. If therefore v and v' denote the angles of deflection in these positions, we have

$$2p = \tan v + \tan v'; \qquad (4)$$

and dividing (3) by this,

$$\tan \theta = \frac{\tan u + \tan u'}{\tan v + \tan v'}. \qquad (5)$$

Thus, from the deflections produced in these four positions of the bar, we obtain the inclination.

In order to determine the *changes* of the inclination, it is not necessary to observe the deflections in the horizontal position of the bar. Let equation (1) be differentiated, X, Y, and u being all variable, and let the resulting equation be divided by (3). We thus obtain the following equation, from which p and q are both eliminated:

$$\frac{\Delta \text{Y}}{\text{Y}} = \frac{2\,\Delta u}{\cos^2 u(\tan u + \tan u')} + \frac{2\tan u}{\tan u + \tan u'} \cdot \frac{\Delta \text{X}}{\text{X}}.$$

But from the relation $\text{Y} = \text{X}\tan\theta$, we have

$$\frac{\Delta \text{Y}}{\text{Y}} = \frac{\Delta \text{X}}{\text{X}} + \frac{\Delta\theta}{\sin\theta\cos\theta};$$

and substituting,

$$\frac{\Delta\theta}{\sin 2\theta} = \frac{\cos u'\Delta u}{\cos u\sin(u + u')} + \frac{1}{2}\frac{\sin(u - u')}{\sin(u + u')}\frac{\Delta \text{X}}{\text{X}}. \qquad (6)$$

The second term of the right-hand member of this equation contains a correction required for the simultaneous changes of the horizontal intensity; but this correction will be generally small, and, when the bar has no permanent magnetism, will vanish altogether. In this latter case, in fact, it appears from (1) and (2) that $u' = u$; so that the preceding equation is reduced to

$$\Delta\theta = \frac{\sin 2\theta}{\sin 2u}\,\Delta u. \qquad (7)$$

We must remember that the angle u in the preceding formulæ, being the deviation of the suspended magnet from the position which it would assume under the action of the earth alone, its changes are the *differences* between the observed changes of position of the suspended magnet, and the *corresponding changes of declination.* Let a denote the deviation of the suspended magnet, measured from some *fixed line,* and a' the corresponding angle when the iron bar is removed; then

$$u = a - a', \quad \Delta u = \Delta a - \Delta a'.$$

But $\Delta a = kn$, $\Delta a' = k'n'$; in which n denotes the number of divisions of the scale of the instrument corresponding to the angle Δa, n' the number corresponding to the angle $\Delta a'$, as shown by the declinometer, and k and k' the arc-values of a single division in each instrument. Hence

$$\Delta u = kn - k'n'. \qquad (8)$$

I now proceed to the construction of the apparatus employed in these measurements.

The magnet is cylindrical; its length is three inches, and diameter one-fourth of an inch. A mirror is attached to the stirrup by which it is suspended, by means of which the varying position of the magnet may be observed with a telescope at a distance, after the method of Gauss. This mirror is of course vertical; and it has a motion round a vertical axis, by means of which it may be adjusted to any desired position of the observing telescope. The mirror is circular, and is three-fourths of an inch in diameter. The moveable part of the stirrup to which it is attached has the form of a cross; and it is rendered vertical by means of three screws, near the extremities of three of the arms of the cross, the heads of which project and hold it. The mirror is maintained in contact with these heads by springs at the back.

The box is octagonal; the interval between the opposite sides is four inches, and that between the top and bottom two inches. The top and bottom, and the connecting pillars, are formed of gun-metal; the eight sides are closed by moveable pieces, three of which are of glass, and the rest of ebony. To the top of the box is attached an upright tube of glass, eight inches in length, which encloses the suspension thread. The suspension apparatus at the top of the tube is of the usual construction; the circular piece to which it is attached has a movement of rotation, and its outer surface is graduated to 5°, for the purpose of determining the effect of torsion of the suspension thread.

The base of the instrument is a circle of gun-metal, six inches in diameter, graduated on the edge. The box is connected with this circle by a short conical stem, forming the axis of a second plate, which revolves upon the fixed one. This moveable plate carries two verniers, by which the angle of rotation may be read off to minutes. Two tubular arms, slightly inclined to one another, are attached to this plate; and their other extremities are connected by a cross-piece, which carries a short scale at a distance of eighteen inches from the mirror. This part of the apparatus is employed in determining the total angles of deflection.

The soft iron bar is a cylinder, twelve inches long, and three-fourths of an inch in diameter. One of its extremities is enclosed in a hollow cylinder of brass, connected with a horizontal pivot which revolves in a fixed socket. The axis of this pivot being in the line passing through the centre of the suspended magnet, and perpendicular to the magnetic meridian, it is obvious that the bar has a movement of rotation in the plane of the magnetic meridian itself. The distance of the axis of the bar from the centre of the magnet is about five inches; and it is so placed that the induced pole is in the direction of the axis of the pivot, and thus remains fixed during the movement of the bar.

The changes of position of the suspended magnet are observed at a distance by means of a fixed telescope and scale. The scale, whose divisions are reflected by the mirror, *is attached above the telescope to the support near the* eye-end.

Dr. Fulton made some observations on Grecian and Roman Architecture.

February 14.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

Captain Stirling, 73rd Regt., Rev. Thomas Stack, F.T.C.D., Joseph Nelson, Esq., Q.C., and Rev. Robert Chatto, were elected members of the Academy.

Dr. Anster read a paper, by the Rev. J. Wills, upon Mr. Stewart's attempt to explain certain processes of the